The Utility of Bayesian Inference in Instrumental Variables Models

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Example: Randomized Experiment with Noncompliance, Sommer and Zeger Vitamin A Data

Row	True Compliance Type	Treatment Assignment	Treatment Received	Y_{obs}	Number of Children
1	?	0	0	0	11514
2	?	0	0	1	74
3	N	1	0	0	2385
4	N	1	0	1	34
5	C	1	1	0	9663
6	C	1	1	1	12
					23682

Reference: Sommer and Zeger (1991). On Estimating Efficacy from Clinical Trials. Statistics in Medicine.

Results of Three Standard MoM Analyses & IVE

Method	Estimate	Calculation	Row Comparison
ITT	-0.0026	$=\frac{12+34}{9663+2385+12+34}-\frac{74}{11514+74}$	3, 4, 5, & 6 vs. 1 & 2
As-treated	-0.0065	$=\frac{12}{9663+12}-\frac{34+74}{11514+2385+34+74}$	5 & 6 vs. 1, 2, 3, &4
Per protocol	-0.0052	$=\frac{12}{9663+12}-\frac{74}{11514+74}$	5 & 6 vs. 1 & 2
Instrumental Variable	-0.0031	$=\frac{\text{ITT}}{\text{Proportion(Compliers)}}$	Requires Exclusion Restriction

Imbens G.W. and Rubin D.B. (1997) Bayesian Inference for Causal Effects in Randomized Experiments with Noncompliance. Annals of Statistics 25(1):305-327.

IVE: MoM CACE Analysis

$\text{ACE} = p_N \cdot \text{NACE} + p_C \cdot \text{CACE}$

$-0.0025 = 0.2 \cdot \text{NACE} + 0.8 \cdot \text{CACE}$

$-0.0025 = 0.8 \cdot \text{CACE} \Rightarrow \text{CACE} = -0.0025/0.8 = -0.0031$

Bayesian Analysis of Sommer & Zeger Data Posterior Distribution of CACE

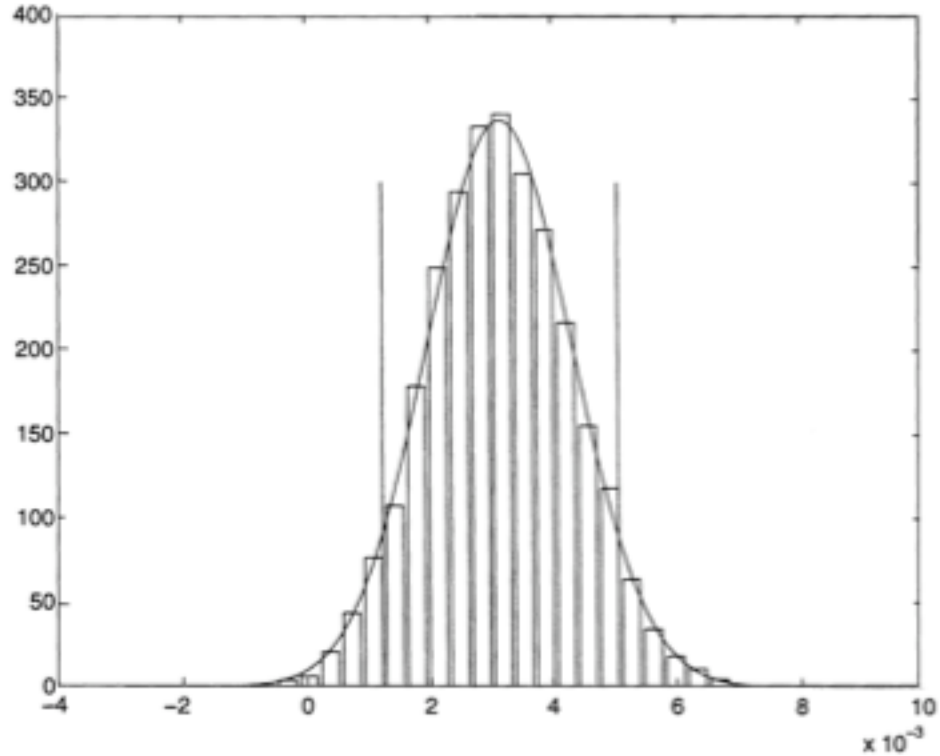

FIG. 3. *Histogram of CACE with exclusion restriction (data from Table 3).*

This Bayesian analysis considers true compliance type to be missing data for people assigned control, and multiply imputes them using observed outcomes.

Bayesian Analysis of Sommer & Zeger Data, Marginal Posterior Distributions with and without Exclusion Restriction

Estimand	Exclusion restriction	Mean	Standard deviation	Median	5th percentile	95th percentile
CACE	No	3.1	2.5	3.2	-0.9	7.0
$ITT_Y^{(n)}$	No	0.5	10.1	0.2	-14.1	17.5
CACE	Yes	3.1	1.2	3.1	1.2	5.1

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Bayesian Analysis of Sommer & Zeger Data

Marginal Posterior Distribution of CACE

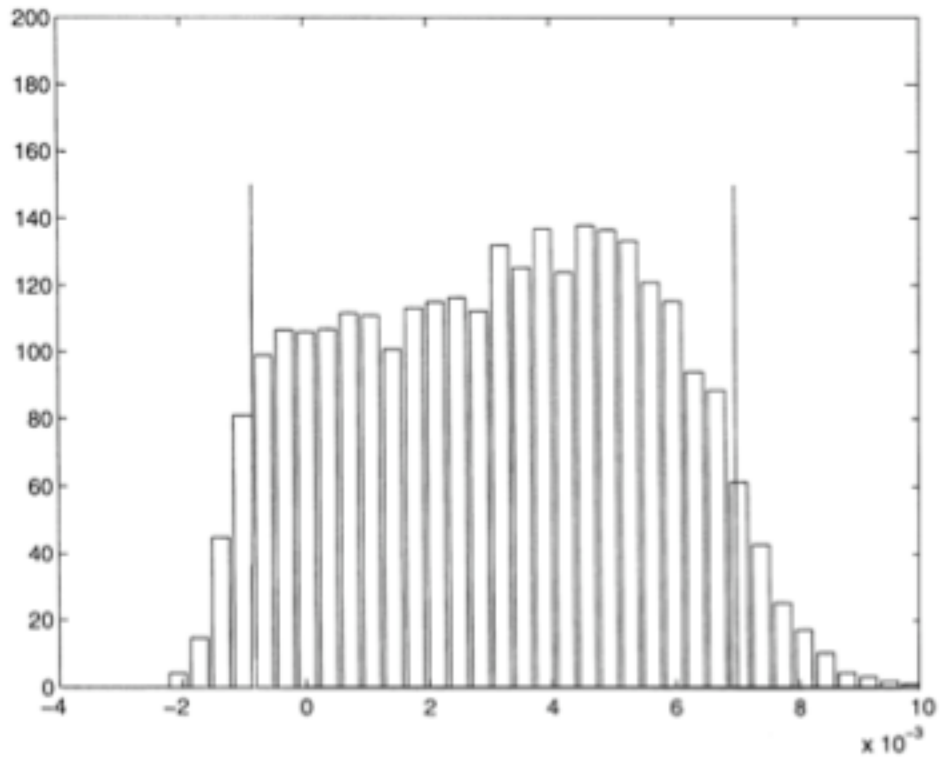

FIG. 1. *Histogram of CACE without exclusion restriction (data from Table 3).*

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Bayesian Analysis of Sommer & Zeger Data

Marginal Posterior Distribution of $ITT_Y^{(n)}$ Without Exclusion

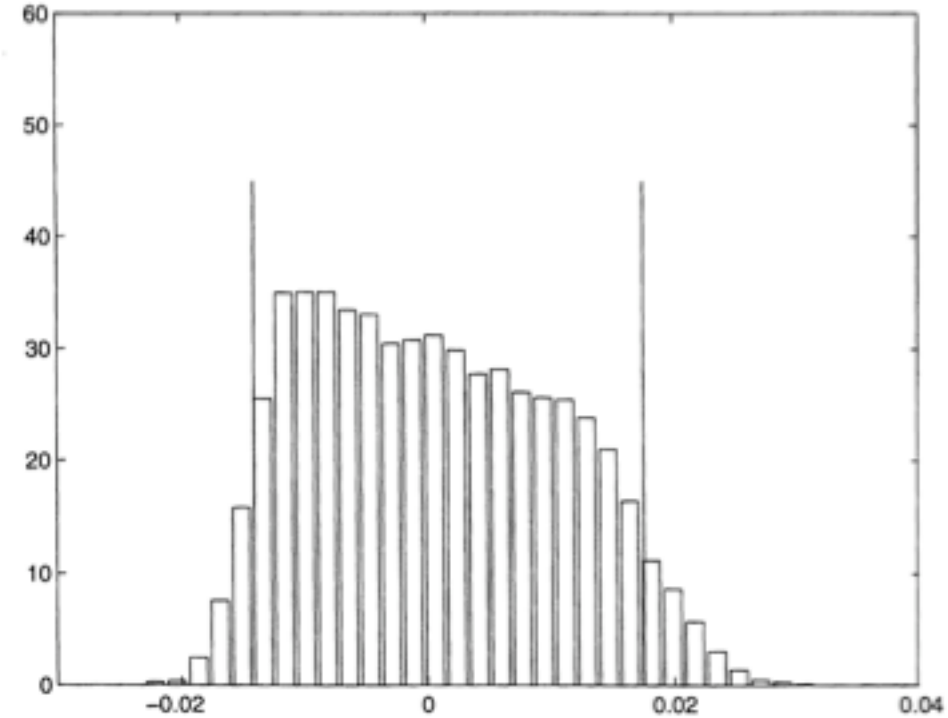

FIG. 2. *Histogram of* $\mathrm{ITT}_Y^{(n)}$ *without exclusion restriction (data from Table 3).*

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Bayesian Analysis of Sommer & Zeger Data

Joint Posterior Distribution of CACE and $ITT_Y^{(n)}$

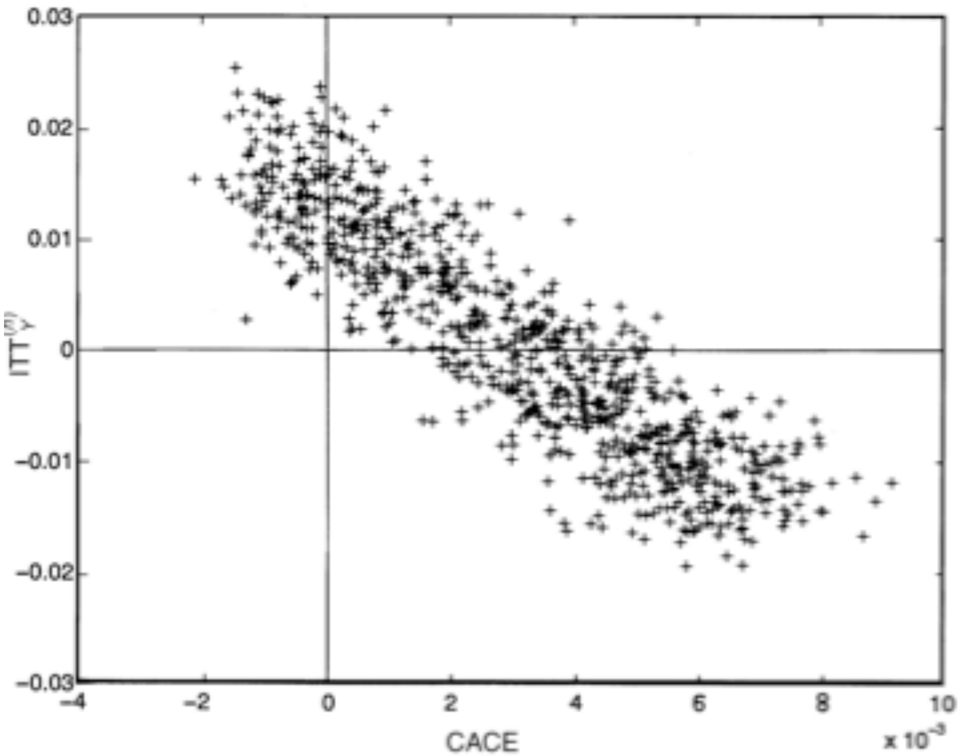

FIG. 4. *Joint posterior distribution of CACE and* $\mathrm{ITT}_Y^{(n)}$ *(data from Table 3).*

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Hypothetical Example Illustrating Frequentist Superiority of Bayes over IVE (and MLE), for CACE under both Exclusion Restrictions and Monotonicity; Z_i is treatment assignment

C_i	$P(C_i \mid \pi)$	$Y_i \mid C_i, Z_i = 0, \pi$	$Y_i \mid C_i, Z_i = 1, \pi$
True complier	0.25	$N(0.1, 0.16)$	$N(0.9, 0.49)$
Never taker	0.45	$N(1.0, 0.25)$	$N(1.0, 0.25)$
Always taker	0.30	$N(0.0, 0.36)$	$N(0.0, 0.36)$

From *True complier* row, CACE = 0.9 – 0.1 = 0.8
Monotonicity means there are no defiers
Exclusion restrictions: no effect of Z_i on Y_i

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One Sample Illustrating Bayes, Standard IVE and Standard MLE-based Methods

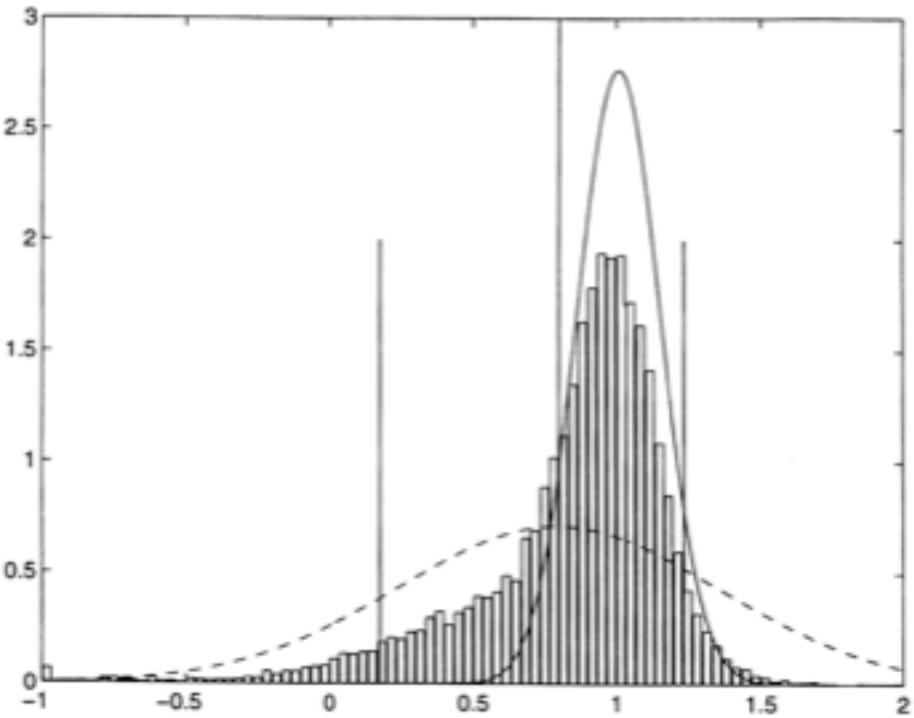

FIG. 5. *Estimates of the posterior distribution of CACE under exclusion restriction and monotonicity condition (data analyzed in Table 6): histogram is based on simulation, solid line is normal approximation based on mle, dashed line is normal approximation based on* $\widehat{\text{IVE}}$.

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Hypothetical Example Illustrating Frequentist Superiority of Bayes over IVE (MoM) and MLE, Frequentist Evaluation under Monotonicity and Exclusion Restrictions

					90% interval	
Estimator	Mean bias	Median bias	Root mean squared error	Median absolute error	Coverage rate	Median width
Posterior mean	-0.10	-0.07	0.48	0.30	0.91	1.61
Posterior median	-0.08	-0.06	0.51	0.32		
MLE	-0.14	-0.12	0.51	0.31	0.74	1.11
IVE	0.55	0.13	2.31	0.54	0.91	2.78

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There exists a variety of more advanced examples of Bayesian superiority.